Invited Lecture

Duos of Artefacts, A Model to Study the Intertwining of Tangible and Digital Tools in Mathematic

Soury-lavergne Sophie[1]

ABSTRACT A variety of tangible manipulatives and digital environments are commonly used in mathematics education. Instead of comparing and opposing the two types of artefacts, we propose to study their combination, with a simplified model of just two artefacts. We define duo of artefacts as a specific combination of complementarities, redundancies and antagonisms between a tangible artefact and a digital one in a didactic situation (Soury-Lavergne, 2021). A duo is designed to provoke a joint instrumental genesis regarding both artefacts, and to control some of the schemes and mathematical conceptualizations developed by pupils during its use. Learning is described in terms of evolution of conceptions in the sense of Balacheff (2013). This lecture illustrates the model with two examples of duo of artefacts for primary school, one in arithmetic and one in geometry. We argue that in addition to be a research tool, duos of artefacts are also a way to support the integration of technology into teachers' practices.

This lecture is about the intertwining of digital and tangible artefacts when manipulatives are introduced in the situations with technologies. My work has its origins in a collaboration with my esteemed Italian colleague, Michela Maschietto, who is in charge of the laboratory of mathematic machines in Modena. In 2010, I was working on technologies, especially dynamic geometry, and she introduced me to some mechanical machines for problem solving in geometry and arithmetic. We began to work as a duo of persons before elaborating together the idea of duo of artefacts. My research question is about how to design and to provide students and teachers with digital technologies and didactical situations using these technologies, that would generate meaningful uses regarding the learning of mathematics and also that could be appropriated by the teachers and be integrated into their practices. Since the eighties and the emergence of personal computers in education, the problem has not been solved. The idea of "duo of artefacts" is a proposal to tackle the question.

This paper present first some assumptions that ground my thinking about technology and the learning of mathematics, which explain how I came to the idea of focusing on the articulation of tangible and digital technologies. Then, I will present in detail the combination of digital and tangible artifacts that constitute a duo of artifacts, which is a model to study systems of instruments. I will illustrate it by two examples, one in geometry, and one in arithmetic. The conclusion raises the main characteristics of the model of duo of artefacts that may be used both for

[1] Institut Français de l'Éducation ENS de Lyon, Laboratoire S2HEP Université de Lyon Université Grenoble Alpes. France. Email: sophie.soury-lavergne@ens-lyon.fr

research purposes as well as for providing teachers and students enhanced learning environments. The whole lecture is available on video at the following url: https://videos.univ-grenoble-alpes.fr/video/20249-icme14-invited-lecture-s-soury-lavergne/

1. Rationale for Studying the Intertwining of Digital Technologies with Tangible Tools

There is currently a growing interest for studying the respective contributions of digital and tangible tools in the learning and teaching of mathematics, like the recent two special issues of the journal Digital Experiences in mathematics Education as shown. In their introduction, Nathalie Sinclair and Richard Nemirowsky raised a lot of questions: “How do the learning affordances of digital and tangible tools differ from each other? Are there optimal combinations of digital and tangible tools? Are there sequences for their alternate use that appear to enhance learning experiences? What theoretical frameworks can help us understand their differences and complementarities?” (Nemirovsky and Sinclair, 2020, p. 108). Duo of artefacts is an attempt to provide first answers to these questions.

1.1. A still lasting concern: integration of technologies in teaching practices

In our community of research about technology, we have a huge concern with is the still low level of integration of technologies in teaching practices, especially in primary school. In 2006, Michele Artigue, who at this time was the vice-president of ICMI, before becoming the president from some years, has concluded the ICMI Study on technologies with a sentence that is still true today: “successful technological integration at large scale level is still a major problem, and this seems to be a general phenomenon” (Artigue, 2010, p. 472).

The work of Ghislaine Gueudet and Luc Trouche (2009) has pointed out the critical role of teaching resources in teachers’ practices. The resources of the teachers have been used as a means to characterize teacher expertise (Pepin et al., 2017; Gitirana et al., 2018). In the case of primary school teachers, there is a significant role of manipulatives in the resources of the teachers. So, a way to perhaps provoke an evolution in the integration of technology in teaching practices for primary school teachers is to build on their existing system of resources. Thus, we have to take into account the presence of manipulatives in those systems, and to understand how technology can be integrated and intertwined to existing systems of resources. It may help to understand and support the integration of technologies into teacher’ practices.

1.2. Questioning the respective role of digital and tangible entities in math learning

Since the first usage of computer in math education, its relationship to tangible objects has been questioned. For Seymour Papert, both computers and tangible objects are a means to carry powerful mathematic ideas into the mind. In the introduction of his famous book “Mindstorms: Children, Computers, and Powerful Ideas” (Papert, 1980), he took the example of the gears of his childhood to explain the double relationship of these gears to the abstract and to the senses. For him, the gears are connected to the formal knowledge of mathematics, and also to what he called the “body knowledge” the sensorimotor schemata of the child. In the nineties, with Sherry Turkle (1992), they claim that “connecting abstract mathematical ideas to the senses” is a characteristic of computers. So, following Turkle and Papert, we may see the computers as a means to replace tangible objects in this idea of making abstract mathematical ideas more concrete.

Later, the definition of virtual manipulatives, that Lauren Resnick and her colleagues (1998) and then Patricia Moyer-Packenham gave (2016), is only centered on digital aspects and digital artefacts. But they still point a relationship with the physical aspects, saying that the important thing is not the fidelity to physical object but the fidelity of the dynamic behavior of the digital object to the mathematical concept. The work of Andrew Manches and his colleagues (2010) compares the use of digital manipulative to the use of tangible manipulatives in problem solving strategies. They have shown that each kind of manipulatives have specific aspects that are important, that guide the mathematical ideas developed by the children and that constraint the strategies pupils can develop in different ways. But there is no evidence for one being better than the other. Their conclusion is that we have to take advantage of the respective contribution of tangible and digital manipulatives.

So, the issue is in the combination of tangible and digital manipulatives to construct knowledge. Which is also the conclusion of Julie Sarama and Douglas Clements (2016), in the book of Patricia Moyer-Packenham about digital manipulatives.

1.3. Theoretical frameworks for mathematical knowledge and tools

Theoretical frameworks for the study of mathematical knowledge and tools have been developed in France. I will first begin with the theory of conceptual fields, from Gerard Vergnaud (2009). I want to precise that Gérard Vergnaud is one of the fathers of the French didactic of mathematics, which passed away a few weeks before the ICME conference, saddening the whole French community of research in math education. For Vergnaud, the root of mathematical conceptualization is in the action. More precisely, the important concept is the concept of schemes, which are the invariant organization of behavior for a class of situations. The epistemic component of schemes, which are the operational invariants, is a key element to understand how knowledge can be built by a user while using tools, either digital of tangible.

The second French theoretical framework is from Nicolas Balacheff (2013), who call it cK¢ for “conception, knowledge and concept”. He is developing the work of Vergnaud, drawing on the definition of concept to elaborate a theory to model mathematical knowledge, that enables to understand "student's understanding". For Nicolas Balacheff, "conception" is modelling every subject way of knowing, be he a student, a teacher or even a mathematician. There is no difference in the nature of their knowledge. The difference lies in the sets of problems they can solve, in the set of representations they manipulate, in the set of operators they are using and also in the kind of control they engage when solving problems.

The instrumental approach of Pierre Rabardel (1995) explains how knowledge develops when using tools. The fundamental distinction made by Rabardel is between artefact and instrument: an instrument is an artefact in situation, associated with usage. When a subject uses an artefact, he develops scheme of use. It is the association of these schemes of use and the part of the artefact that is involved in the schemes that constitutes the instrument. The instrumental genesis synthetizes the conjoint development of schemes and the evolution of the artefact. As Vergnaud explains, there is an epistemic dimension in schemes, with the operational invariants. Indeed, the theoretical distinction between artefact and instrument and the instrumental genesis enable to grasp the construction of knowledge when a subject turns an artefact into an instrument.

1.4. Setting up the problem

The above theoretical frameworks produce two assumptions dealing with the construction of knowledge when using either digital or tangible tools: (i) knowledge (i.e. conception in the sense of Balacheff (2013)) develops through instrumental genesis when using artefacts and (ii) knowledge resides into schemes and their operational invariants.

Therefore, initial questions about the intertwining of digital and tangible artefacts turn into:

- How to design and combine tangible and digital artefacts for the construction of mathematical knowledge by students?
- Does the combination of digital and tangible artefacts help teachers to develop a resources system including technologies?

2. Duo of Artefacts, a Model to Design and Study Systems of Instruments

To answer the previous questions, there is a need for simplification of the learning situation which involves a multiplicity of artefacts in the hand of the students and the teachers. To tackle this complexity, the idea was to reduce it by looking for a simplified model, that is just a pair of artefacts. My purpose is to show that, by carefully choosing the characteristics of the two artefacts we select, we obtain a simple system that is still relevant to study and to understand students' learning and teachers' practices with technology. But there are conditions to be fulfilled for two artifacts in order to form a duo.

A first condition is to have some complementarities between the two artefacts. A very pragmatic solution is to choose one tangible artefact and one digital artefact to obtain complementarities between the two.

2.1. Tangible artefact, digital artefact?

The tangible artefact is defined by its physical properties as an object, like mass, color, movement. It is subject to gravity. It is visible and cannot disappear. It can be manipulated under the physical constraints. In opposition to the tangible one, the representations in the digital artefact are not constrained by the same laws of physics within the user interaction. Nevertheless, digital representations have physical properties too. Indeed, they are produced and embedded in hardware. And they appear to be tangible too, as far as one can operate on it like on tangible objects.

Thus, there is a need for some clarification about the choice of words. I choose to distinguish tangible from digital, even if it is subject to discussion. Some colleagues prefer to use physical artefact instead of tangible, although it must be noted that physical is a property of both kind of artefacts. Some others may choose “concrete” versus “abstract”. But Sarama and Clements (2016) explained that concrete is not only physical: many students, and us also, manipulate mathematical objects, like numbers, as if they were concrete. Concrete results from the connection of these objects with meaningful experiences, which can surely be the case with digital artefacts. Behind each choice of vocabulary, there is the necessity to distinguishes two kinds of artefacts in a continuity of objects and properties. A first demand is that these two artefacts have complementarities.

Our proposal with Michela Maschietto is a very pragmatic one in order to select two artefacts with complementarity. When choosing a tangible artefact on one side and a digital artefact on the other, that is an artefact embedded in a digital environment, we are sure to get both differences and potential complementarities between the two. With the tangible artefact, we get the gestures, the student’ s bodily involvement and the eye-hand sensory-motor loop. With the digital artefact, we get extended possibilities of feedback and some specific behavior adapted to the mathematical knowledge at stake.

2.2. Two artefacts are not necessarily a duo

Even with complementarity, there is no reason that any set of two artefacts may turn into a system of instruments during the instrumental genesis. In fact, each of them can be the object of two separated instrumental genesis, resulting into two independent instruments. For instance, if you consider a calculator to produce the result of an operation and a compass to draw a circle, they are complementary because each of them is adequate to a specific problem but they do not necessarily form a system of instruments. Complementarities will give a purpose to use both artefacts. But two artefacts are not necessarily a duo, resulting in a system of instruments (Fig. 1). Our question is about identifying the characteristics of two artefacts in order to form a system of instruments during the instrumental genesis.

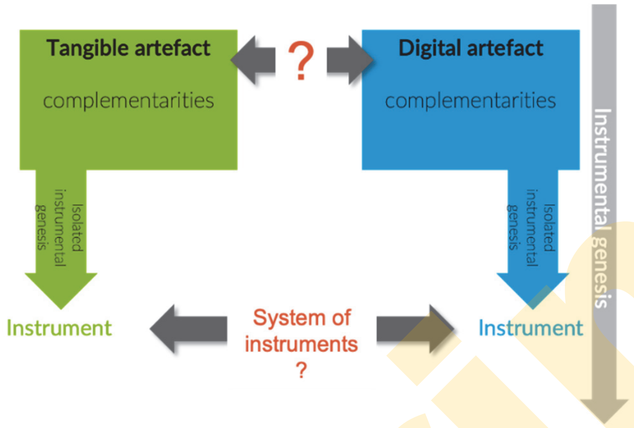

Fig. 1. Two complementary artefacts do not necessarily develop into a system of instruments across an instrumental genesis

Gaetan Bourmaud (2007) has studied systems of instruments in the framework of the instrumental approach. He concludes that there is a need of complementarities but also a need of redundancies between the two artefacts and even some antagonisms. Redundancies in a system of instruments ensure some adaptability, flexibility and robustness of the system. And, more surprising, a last characteristic of a system of instruments is antagonism between the elements. Antagonism seems counter-intuitive. It characterizes functionalities that are present in one bartefact but inefficient or even divergent in the other. Bourmaud explains that a system of instruments is both "(2013) more and less than the sum of the functionalities of each artefact" (2007, p. 65). In a learning perspective, it seems to be very important, since learning is also overcoming some obstacles and adapting a way to solve problem to a new situation. When it is about learning, easing the action is not always the goal. Considering that learning is the evolution of schemes by accommodation and assimilation, using one artefact then another, with the necessity to adapt and to change the solving strategy may be powerful.

2.3. Duo of tangible and digital artefacts and genesis of a system of instruments

Our proposal is to take two artefacts (Fig. 2) that present:

- Complementarities between each other, that will justify the interest to use of both of them, and not only one.
- Redundancies that help user to link the two artefacts and that create robustness and adaptability
- And antagonisms that provoke adaptations, evolutions and finally learning

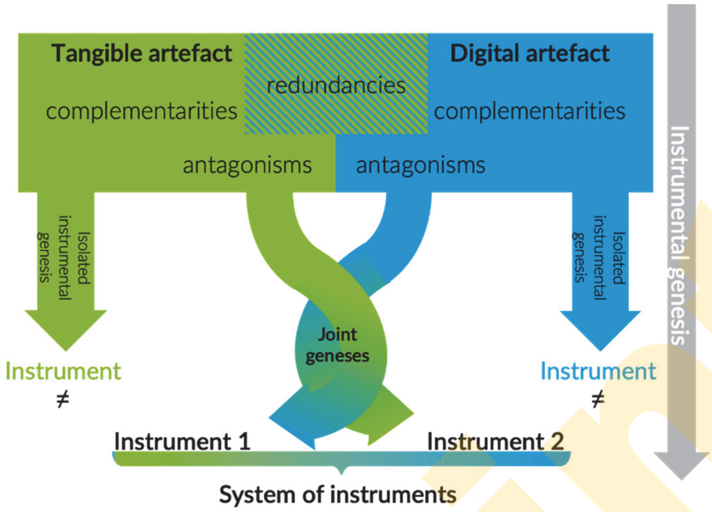

Fig. 1. A duo of artefacts, with complementarities, redundancies and antagonisms may develop into a system of instruments.

But without a purpose to use the artefact, without a task to achieve, there will be no instrumental genesis. It calls for a last characteristic for two artefacts to become a duo to teach or to learn mathematics: a didactical situation. The didactical situation (Brousseau, 1997) brings the purpose of using both artefacts and characterizes the meaning of the knowledge that will be constructed by using them.

3. A First Example of Duo of Artefacts in Geometry: A Duo to Learn How to Construct a Triangle

Anne Voltolini (2018) has elaborated a duo of artefacts to address a difficulty of teacher's practices, which is the construction of a triangle given the length of its three sides.

In French primary schools, the construction with compass is introduced as a very procedural way to construct triangles: draw a first side, open the compass according to the length of the second side , pick the compass on one segment endpoint and draw an arc of circle, open the compass according to the length of the third side, pick the compass on the other segment endpoint and draw a second arc of circle, intersecting the first one, this intersection is the third vertices of the triangle. In consequences, students do not relate the procedure to any geometrical properties. Research in didactics have produced several explanations about the causes of students' difficulties, and thus the difficulties encountered by the teachers. Students' difficulties result at least from three aspects.

First, this construction of the triangle is based on the dimensional deconstruction of geometrical figure (Duval, 2005). Most students see the triangle as a two-dimensional surface. But to succeed in the construction, they have to anticipate a point, the third vertices of the triangle, which is a zero-dimensional object. Thus, the construction requires a dimensional deconstruction from a two-dimensional object to a zero-dimensional object, a point which is rarely conceptualized at this level of education. The second explanation is the fact that the two sides that have to be constructed, once the first side is drawn, are not produced by the compass, which is the tool involved in the construction. The association of the artefact compass to the drawing of straight lines is not direct. The last cause is the fact that the artefact compass is at the origin of different kind of instruments. The main instrument developed by students when using compass is a compass to draw circles. It may also be another instrument which is to transfer lengths. None of them is related to the construction of sides of triangle or construction of points.

3.1. Dynamic segments, compass, and a new students' conception of the triangle

The solution Anne Voltolini has designed is a duo of artefacts associated to a possible new students' conception of the triangle. This new conception is a "line conception", which play the role of intermediary conception between the conception of a triangle as a surface and a conception of the triangle as a set of dots (defined by three vertices). A line conception of the triangle refers to the triangle as a set of line, specifically as a closed broken line of three segments. Voltolini assumed that this line conception would be accessible to primary school students and would help students to understand the construction of a triangle with the compass. To support the genesis of this new conception, she has also identified a new kind of instrument that can be elaborated from the artefact compass. The key idea is that the artefact compass can be an instrument to "rotate segments around one of their endpoints". According to these two hypotheses, she has built a didactical situation, in which students have to use dynamic segments to form a broken line at the interface of the computer and then to try to close the broken line in order to obtain a triangle if possible (Fig. 3, see in the next page).

The problem can also be posed within the paper and pencil environment. A broken line of three segments is drawn and the task is to construct the triangle with the same lengths (Fig. 4, see in the next page). In this situation, compass may appear has a means to rotate segments.

The digital environment allows to create constraints and feedback to bring students to the construction of the triangle. A main constraint is created by the behavior of dynamic segments. Segments cannot rotate and translate simultaneously. Thus, the user has to operate the two movements successively. This separation of the two movements is not possible in the tangible world, for instance when forming a triangle with sticks, because movement of sticks is a combination of translation and rotation. From three separated segments, translating the segments to form a broken line, then rotating the two segments at the extremity to close the broken line is an efficient strategy to form a triangle. This constraint brings the rotation of the segments to the front.

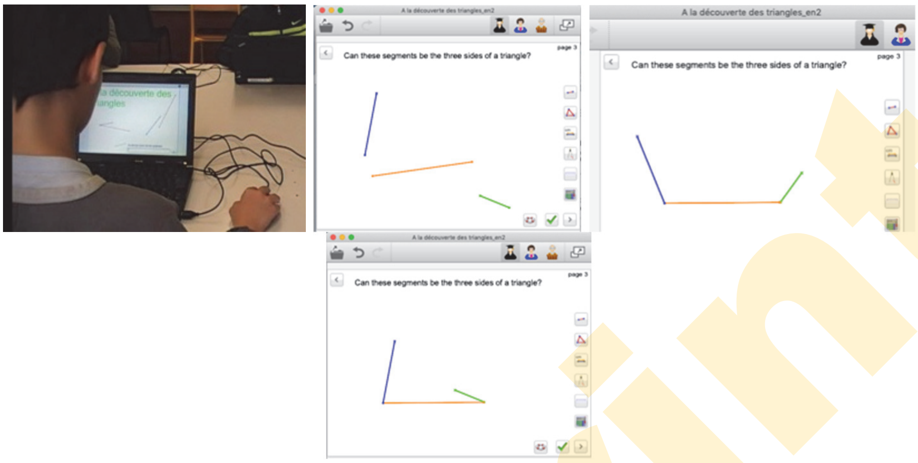

Fig. 2. In the digital environment, a triangle may be formed by translating and rotating dynamic segments.

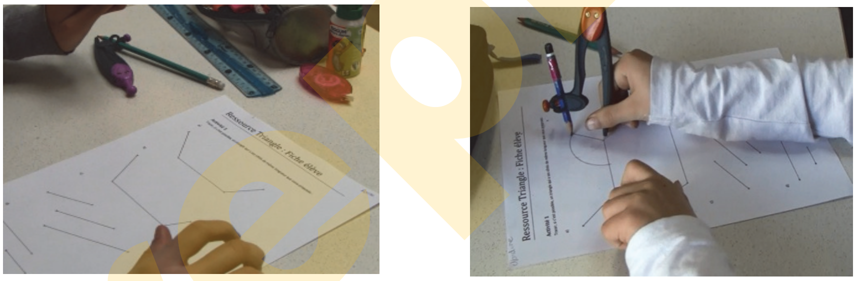

Fig. 4. In the paper and pencil environment, a broken line may be transformed into a triangle with the same segment length by using the compass.

Furthermore, the digital environment gives the possibility to block the movements in order to force students to anticipate and to look for other ways to decide if there will be a triangle or not, leading them to the triangle inequality.

3.2. Analysis of a duo formed by dynamic segments and compass

The analysis of this duo consists first to identify the two artefacts of the duo. On one side there is the dynamic segments, on the other side the tangible artefact which is the compass. There may be a joint genesis because:

- Complementarities between the two artefacts make each of them useful: dynamic segments force the dissociation of the two movements and the compass emphasis the trace of the endpoints of the segments.

- Redundancies help user to link the two artefacts, create robustness and adaptability: both environments focus on rotation and on segments endpoints. But this is a rather low level of redundancies, not very explicit at first glance for the users.
- Antagonisms provoke adaptations, evolutions and finally learning: from the digital environment to the paper and pencil environment, the segments are not dynamic anymore. When they encounter the paper and pencil task, students have to develop a new way to close the broken line and to get the triangle. They spontaneously call upon the compass.

3.3. Evolution of 5-grade students' conception about triangles

Anne Voltolini has conducted an experimentation with two teachers over three consecutive years, with a methodology of design-based research (Coob et al., 2003). Her aim was to observed the evolution of the students' conceptions about triangles and the emergence of a new instrument related to the compass.

She has analyzed in detail the work of 34 pupils using her duo of artefacts. She has observed students activity within the digital and the paper-pencil environments and analyzed their gestures and behavior (Fig. 5) in order to characterize the components of their conceptions about triangle and the characteristics of their instrumental genesis related to the compass. The results show that 30 of the 34 students evolved toward the one-dimensional conception of triangle, which is a very encouraging result. But only five of the students reach the dot conception of a triangle and three students stayed to their initial conception of the triangle like a surface (Fig. 6). About the compass as an instrument to rotate segments, 31 students built this instrument and half of them could identify the circle as a geometrical object involved in the construction of triangles (Fig. 6).

Fig. 5. Student's hands rotation to show the segments movement to form a triangle, as an indicator of the genesis of the compass as an instrument to rotate segments.

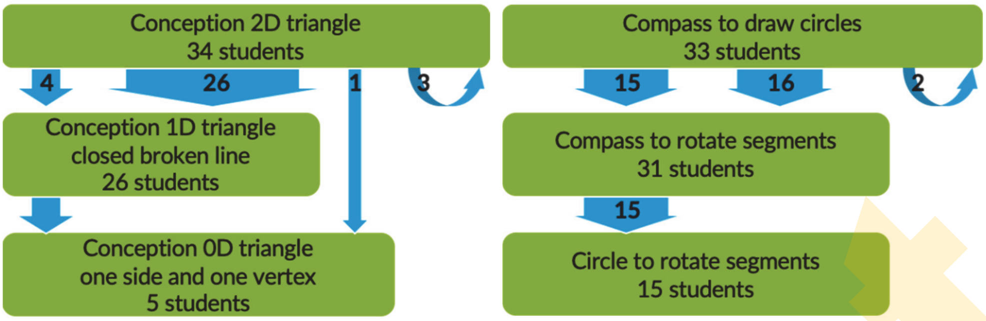

Fig. 3. Evolution of students' conceptions about triangle (on the left) and evolution of the instruments related to the compass (on the right) for 34 students of 5-grade.

3.4. Conclusion about duo of artefacts in geometry to provoke conceptual understanding of triangles and their construction.

This first example of duo of artefacts in geometry provides an example of a joint genesis of a system of instruments from digital and tangible artefacts and its consequence on learning. A first issue in the analysis of a situation with a duo of artefacts is to select the two artefacts of the duo. In this case, the duo is formed by dynamics segments, on one side, associated to a compass on the other side. The redundancy between the two artefacts is not very strong. Nevertheless, the duo enables the genesis a new instrument associated to the artefact compass, that is to turn segments around one endpoint. Furthermore, this new instrument has its roots in the dynamic segments that have been manipulated in the digital artefact.

The second result of Voltolini's work is the conception of triangle as one-dimensional object. It is a way to enact dimensional deconstruction (Duval, 2005) from two dimensions to one dimension, and maybe a step toward a dot conception of triangles. The line conception of triangle is a reachable learning objective, thanks to the duo and the instrument compass to rotate segment. It is clearly expressed by Luna, a young student who links both the triangle and the compass, the knowledge and the instrument: "The broken line, it helps, because, before, you don't know that you have to use compass to draw a triangle" (our translation).

4. A Second Example of Duo of Artefacts in Arithmetic: the Pascaline and E·Pascaline

The second example concerns arithmetic. It is the duo formed by the pascaline and the e·pascaline that we have designed together with Michela Maschietto (Maschietto and Soury-Lavergne, 2013, 2017; Soury-Lavergne and Maschietto, 2015). This example may be a kind of exemplar in the sense of Kuhn (1977), that is a solution to concrete problems, accepted by the group as paradigmatic.

4.1. *Designing the pascaline and e·pascaline duo*

The Pascaline is a small mechanical machine (Fig. 7), to write numbers and to do calculations. When using the pascaline, which is a set of wheels, you click on the wheel to write numbers with the digits displayed on the teeth. There are different kinds of strategies to write numbers, to calculate and to solve problems. Among them, two main procedures. One procedure is by iteration. You use only the unit wheel and you pass the numbers one after the other. You follow the number sequence, one click on the wheel for one number. The other procedure is by decomposition. You use each of the three wheels, to write or to calculate, according to the base-10 place value system to write numbers. The actions of the user on the pascaline are different according to these two procedures, which makes the value of the pascaline for teaching number system.

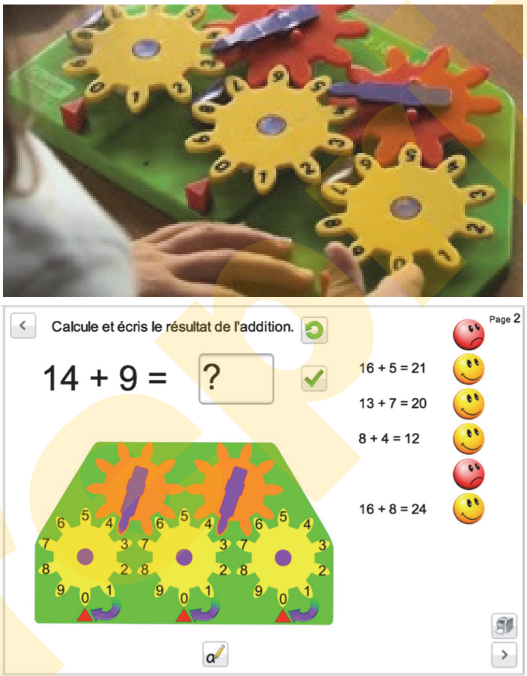

Fig. 4. The pascaline (to the left) and the e·pascaline (to the right) in a duo of artefacts.

The e·pascaline has been designed with the Cabri Elem technology (Fig. 7). The idea was to add features and feedback to the functioning of the pascaline: to enrich the functioning of the pascaline; to emphasize the mathematical properties that are relevant for learning base 10 place value system; but also to minimize the prevalence of some of other properties that may distract from learning at stake.

With Michela Maschietto (Maschietto and Soury-Lavergne, 2013), we choose to preserve some visual fidelity to ensure some redundancies and connection between the tangible Pascaline and the digital counterpart (for instance the colors or the purple arrow, we have used drawing of many pupils to determine the one to keep). Also, we used the digital environment to create additional constraints, that would provoke students' adaptation of their procedure. A main constraint is on the possible actions to turn the wheels: you don't act directly on the wheel but you have to click on a small button, which have the shape of an arrow. Sometime, the arrow disappears, preventing you to make the wheel turning. This behavior should provoke an evolution of the user procedure, especially a transition from the iteration procedure to the decomposition procedure. We have also implemented different kinds of feedback, including an evaluation of the student's answer.

4.2. Complementarities, redundancies and antagonisms in the pascaline and e·pascaline duo

The complementarities of the pascaline and the e·pascaline in the duo lie in the following characteristics. The tangible pascaline produces sound and haptic feedback that are not existing in the e·pascaline. The e·pascaline offers also additional functionalities in comparison to the tangible pascaline, like the reset button to display zero on the three wheels, a counter of clicks which displays how many times you have click on a wheel and an evaluation feedback with smileys, expressing success and failures. There are also redundancies. They concern not only the visual aspect (Fig. 8, see in the next page) but also the two main strategies. The iteration and the decomposition strategies are available in both artefacts, even if not always with the e·pascaline. The antagonisms between the two artefacts concern the possible actions on the artefacts. With the e·pascaline, user action on the wheel may be controlled. Depending on the situation, the action on the wheels can be free or can be restricted or even stopped.

Finally, there is a didactical situation framing the students' activity, to make them learn to write numbers, to calculate, to solve arithmetic problems. The didactical situations are partially embedded in the digital environment. It is not possible to do the same with the tangible pascaline.

4.3. 1st-grade students' conceptions about numbers and base-10 place value system

Like in the domain of geometry, learning is modelled by an evolution of conceptions. Thus, different conceptions about numbers and base-10 place value system must be distinguished to understand how the duo can provoke learning (Soury-Lavergne and Maschietto, 2015). According to Balacheff (2013), a conception is defined by the description the problems it enables to solve, the operators, the semiotic system to express the operators and the problems, and the types of control.

Following these elements of description, a first conception about numbers consists in seeing numbers as measuring a set of entities. For instance, number 17 is seen as 17 units. Numbers form a sequence; they follow or precede each other. Digits are placed side by side to form an iconic writing of a number and each number has a proper name, without a necessary relation with the other names of numbers. Adding is "counting on" and the decomposition of numbers (like the decomposition of 8 into 3 and 5) is not seen as a calculation. Another conception is embedding the previous one and a first view on base-10 place-value system. It means that number measures an organized set of entities, grouped in 10 and some units. The number code obeys to principles in order to deduce mathematical properties, like $17 = 10 + 7 = 1\mathrm{t} + 7\mathrm{u}$. Adding is using the information given by the digit in the number code and the sequence of number is seen as generated by adding one unit. The aim of teaching with the duo at primary school will be to make students evolve from one conception to the other.

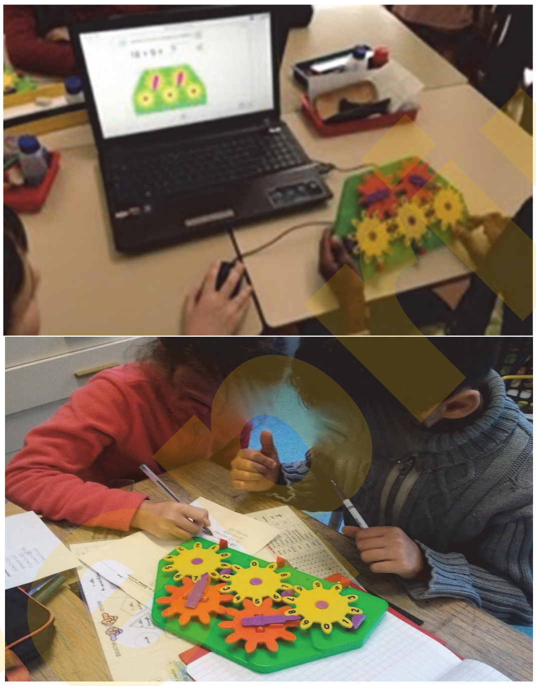

Fig. 5. Students activity with the duo of artefacts pascaline and e·pascaline

4.4. Learning with the pascaline and e·pascaline duo, the case of addition

We have conducted several observations of classes, in different contexts in particular in two classes of 6-year-old pupils in France. I will take the example of addition with the pascaline and the e·pascaline to illustrate the change in students' conception.

With the pascaline, iteration and decomposition procedure are possible. We have observed that students do not use the iteration procedure spontaneously. But once they discover it, they use intensively the iteration procedure even with large numbers over one hundred. In fact, they show their expertise in mathematics by counting numbers one by one. The iteration procedure did not take too long and did not generate enough errors to lead pupils to look for another procedure. The e·pascaline works similarly to the pascaline. But, with the e·pascaline, the new constraint on action forces students to drop out the iteration procedure.

If you use the e·pascaline, you will experiment that if you want to add 14+16, you can first display 14 on the wheels, and then begin to add 16 by clicking 16 times on the unit wheel. But after some clicks, you will be stopped because the action button disappears. It creates a problem-solving situation for first grade students. They couldn't find out how to complete the calculation with the e·pascaline. In one of the class, some students have asked to use the pascaline to perform the calculation. One they obtain the result, with the pascaline, they wrote the result on the e·pascaline to get the evaluation feedback. They couldn't mobilize another strategy, which show the difficulty to consider numbers through their numeral writing and to exploit the base-10 place value system.

The problem couldn't be overcome without the teacher intervention. In one class, teacher intervention introduces the decomposition procedure. In the other class, the teacher took in charge the evolution of the procedure in a different way. She first brought the students to a common statement: we are blocked. Then she has formulated the problem: how to overcome the limitation of the use of the unit wheel? Finally, she gave pupils a hint by asking them to look for different additive decompositions of numbers. One of those decompositions is 16 as being 10+6 and with the conversion of 10 units into one ten, the problem could be solved.

In conclusion, the feedback helps to bring the problem to the front, but it is not enough to make pupils change procedure. These first studies have confirmed the great resistance of pupils to abandon iterative strategies in favor of decomposition strategy. It also reveals a conceptualization of number that does not yet incorporate the principles of number decimal writing. Nevertheless, it offers students and teachers a field of experiences that distinguishes two ways to operate with numbers that they can bring to the discussion.

4.5. Teaching with the pascaline and e·pascaline duo

Our aim with duo of artefacts is not only providing students with new environment and new opportunities to learn mathematics, but also to help teachers to integrate technology into their system of resources. So, we have conducted an experiment, with 8 voluntary teachers of first grade which were note involved in the design of the duo, to understand if the use of a duo of artifacts is possible and the condition of success in its integration (Maschietto and Soury-Lavergne, 2017).

The experiment has been conducted in France over a period of 12 weeks. We have made some direct observations, we have received regular reports from the teachers, including students' productions, and we have interviewed the teachers on a regular basis. Our question was: Could teachers integrate the duo in their system of resources?

Our criteria to evaluate the integration of the duo by teachers were: (i) the creation of new resources and new situations of use of the duo of artefacts in their class; (ii) the actual organization of a class spatial/temporal configuration that enable to give students access to both artefacts; (iii) the awareness of some didactical aspects of the digital and tangible use of the duo.

One teacher left the experiment for medical reason, but the 7 other teachers have developed a wide range of ways to use the duo. First, they have implemented various didactical configurations, with the use of the pascaline or the e·pascaline, in collective setting, or with pair of students, or with student using individually the pascaline or the e·pascaline. They have also combined simultaneous use of the pascaline with the e·pascaline, for instance, every student has a pascaline and the e·pascaline is displayed on the wall (Fig. 9). There is also successive use, first the pascaline then the e·pascaline or conversely, the e·pascaline is first collectively used in class, then students work individually with their pascaline.

Fig. 9. A 1st grade class using simultaneously the pascaline (on students' desks) and the e·pascaline displayed on the wall.

The teachers have produced additional resources and situations to those provided by the research team (Soury-Lavergne, 2014). However, one very important point is that they begin to be aware of the students' strategies and they tried to act and control it by using one or the other artefact. For instance, when students were blocked with the e·pascaline because they wanted to use the iterative strategy, one of the teachers gave access to the tangible pascaline, to help the student. By doing so, she has modified the didactical situation and allowed the pupils to solve the problem without having to change their procedure. It may be a problem for the learning, if the evolution toward the decomposition procedure never occurred.

Nevertheless, it reveals that the teacher played with the didactical characteristics of the duo. Therefore, the duo of artefacts became a system of instruments in the hand of the teachers. They have developed ways to exploit the complementarities and the antagonisms of the two artefacts of the duo. In conclusion, there are evidences of appropriation of the duo by teachers, to teach arithmetic's and to look for conceptual understanding, which means that there is an instrumental genesis of the duo of artefacts among every of the seven teachers.

5. Conclusion, duos of tangible and digital artefacts, a means to study learning and teaching with technology

As a conclusion, I will first address the question of "why designing a duo of artefacts?" before dealing with "how to design it" and "would it help to teach and learn mathematics?"

A first reason to design a duo is because it provides students with a rich learning experience. The two examples of duo, one in geometry and one in arithmetic, have demonstrate that using a duo of artefacts helps students to develop new conceptions about mathematical concepts. Like a one-dimensional conception of the triangle associated to the compass to turn segments or to gain new insight in base-10 place value system for numeral writing, as a tool to solve problem. It is also a way to support dissemination and actual appropriation of digital technology by teachers. In our experiment, we have obtained evidence that it is possible. Teachers became aware of the complementarities of digital technology regarding the already used manipulatives. A duo also provided them with flexible configurations and possibilities for adaptation. A last point in concerning the involvement of teachers into research projects. Involving teachers in the design of a duo of artefacts appears to be efficient to enroll teachers in research. Teachers and researchers can share an initial aim and build collaboration on each other expertise.

The design of a duo of tangible and digital artefacts constitutes the main part of this lecture. The first point is to identify or to create two artefacts, with complementarities, redundancies and antagonisms. A pragmatic choice is to take a tangible and a digital artefact, which is an easy way to ensure complementarities, redundancies and antagonisms between the two. But it is not a necessity. The tangible artefact brings haptic feedback and gesture which are not so easy to obtain with digital artefact. The digital artefact brings feedback about procedures and evaluation which are also not easy to obtain with tangible manipulatives, which have a lot of limitations. Finally, a critical point is the elaboration of a didactical situation, that require the use of both artefacts, and initiate the instrumental genesis. The didactical analysis which is behind the identification of student conception and their evolution thanks the duo of artefacts is also a critical point.

The idea of duo of artefacts raises also new questions for research. Some are related to the model: for instance, how to decide which are the artefacts of the duo to be considered in a given situation to produce the analysis? And a more general question: which duo of tangible and digital artefact for a given mathematical knowledge? Currently we do not have a stabilized technique to design a duo of artifacts. But it seems to be relevant in a lot of different situations. Several researchers have already considered the combination of digital and tangible artefacts in their own work for different pieces of knowledge. Duos of artefacts are a tool to focus the analysis on the intertwining of tangible and digital tools in mathematics.

References

M. Artigue (2010). The Future of Teaching and Learning Mathematics with Digital Technologies. In C. Hoyles and J.-B. Lagrange (Éds.), *Mathematics Education and Technology-Rethinking the Terrain* (Vol. 13, p. 463–475). Springer US.

N. Balacheff (2013). CK¢, a model to reason on learners' conceptions. In M. V. Martinez & A. Castro Superfine (Éds.), *PME-NA Psychology of Mathematics Education North America Chapter*

G. Bourmaud (2007). L'organisation systémique des instruments : Méthodes d'analyse, propriétés et perspectives de conception ouvertes. *Colloque de l'Association pour la Recherche Cognitive — ARCo'07 : Cognition — Complexité — Collectif*, 61–76.

G. Brousseau (1997). *Theory of Didactical Situations in Mathematics*. Kluwer Academic Publishers.

P. Coob, J. Confrey, A. diSessa, and L. Schauble (2003). Design Experiments in Educational Research. *Educational Researcher*, 32(1), 9–13.

R. Duval (2005). Les conditions cognitives de l'apprentissage de la géométrie : Développement de la visualisation, différenciation des raisonnements et coordination de leurs fonctionnements. *Annales de didactique et de sciences cognitives*, 10, 5–53.

V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, and L. Trouche, L. (Éds.). (2018). Understanding Teachers' work through their interactions with resources for teaching. In *Res(s)ources 2018 International Conference*.

G. Gueudet and L. Trouche (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71, 199–218.

T. S. Kuhn (1977). *The Essential Tension Selected Studies in Scientific Tradition and Change*. University of Chicago Press.

A. Manches, C. O'Malley, and S. Benford (2010). The role of physical representations in solving number problems: A comparison of young children's use of physical and virtual materials. *Computers & Education*, 54, 622–640.

M. Maschietto and S. Soury-Lavergne (2013). Designing a duo of material and digital artifacts: The pascaline and Cabri Elem e-books in primary school mathematics. *ZDM*, 45(7), 959–971.

M. Maschietto and S. Soury-Lavergne (2017). The duo "pascaline and e-pascaline": An example of using material and digital artefacts at primary school. In E. Faggiano, F. Ferrara, & A. Montone (Éds.), *Innovation and technology Enhancing Mathematics Education* (Springer, p. 137–160).

P. Moyer-Packenham (Éd.). (2016). *International Perspectives on Teaching and Learning Mathematics with Virtual Manipulatives*. Springer Berlin Heidelberg.

R. Nemirovsky and N. Sinclair, N (2020). On the Intertwined Contributions of Physical and Digital Tools for the Teaching and Learning of Mathematics. *Digital Experiences in Mathematics Education*, 6(2), 107–108.

S. Papert (1980). *Mindstorms. Children, Computers, and Powerful Ideas.* New York: Basic Books.

B. Pepin, B. Xu, L. Trouche, and C. Wang (2017). Developing a deeper understanding of mathematics teaching expertise: An examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics*, 94(3), 257–274.

P. Rabardel (1995). *Les Hommes & Les Technologies : Approche Cognitive Des Instruments Contemporains*. Armand Colin.

M. Resnick, F. Martin, R. Berg, R. Borovoy, V. Colella, K. Kramer, and B. Silverman (1998). Digital manipulatives: New toys to think with. *CHI ‘98*, 281–287.

J. Sarama and D. H. Clements (2016). Physical and Virtual Manipulatives: What Is “Concrete”? In P. S. Moyer-Packenham (Éd.), *International Perspectives on Teaching and Learning Mathematics with Virtual Manipulatives* (Vol. 7, p. 71–93). Springer International Publishing.

S. Soury-Lavergne (2014). *MOM pascaline et e-pascaline, une Mallette d’Outils Mathématiques pour la numération et le calcul en CP*. Institut Français de l’Education.

Soury-Lavergne, S. (2021). Duos of Digital and Tangible Artefacts in Didactical Situations. *Digital Experiences in Mathematics Education*, 7(1), 1–21.

S. Soury-Lavergne and M. Maschietto (2015). Number system and computation with a duo of artefacts: The pascaline and the e-pascaline. In X. Sun, B. Kaur, and J. Novotna (Éds.), *The Twenty-third ICMI Study: Primary Mathematics Study on Whole Numbers* (p. 371–378). ICMI.

S. Turkle and S. Papert (1992). Epistemological Pluralism and the Revaluation of the Concrete. *Journal of Mathematical Behavior*, 11(1), 3–33.

G. Vergnaud (2009). The Theory of Conceptual Fields. *Human Development*, 52, 83–94.

A. Voltolini (2018). Duo of digital and material artefacts dedicated to the learning of geometry at primary school. In L. Ball, P. Drijvers, S. Ladel, H.-S. Siller, M. Tabach, and C. Vale (Éds.), *Uses of Technology in Primary and Secondary Mathematics Education: Tools, Topics and Trends* (p. 8–99).